Steady State Creep of a Composite Material with Short Fibres

S. T. MILEIKO*
Institute of Solid State Physics of the Academy of Sciences of the USSR, Moscow, USSR

The shear within a matrix volume is assumed to be an important process during the creep of composite material reinforced with short rigid fibres. The rate of elongation of such a composite with certain fibre distributions can be estimated. The agreement with a few experimental data is reasonably good.

1. Introduction

Various deformation processes of a matrix and fibres can take place during the creep of a fibrous composite. Considering the stability and long-time rupture of such materials one should also take into account processes localised at the fibre-matrix interface. However, for a first approach to a solution of the problem it is certainly possible to disregard the special problem of the interface for two reasons. Firstly, at the present time the physical description of the behaviour of the interface is insufficiently comprehensive to form a base on which to construct a model for calculation. Secondly, it is possible to describe to a first approximation the creep and creep-rupture behaviour of sufficiently stable continuous fibrous composites based only on the properties of a matrix and fibres [1].

The present state of affairs in the problem of the creep of discontinuous fibrous composites, as far as the author is aware, is that there are only the experimental results of Kelly and Tyson [2] and the unpublished work by McLean [3], who analyses the problem taking into account the contribution of the shear of the interface and the diffusion creep of a matrix at the end of a fibre.

It seems to be clear, following the experience of mechanical theories of creep of homogeneous materials, that the only way of developing the engineering theory of fibrous composites that might have real practical application is by building up some combination of more or less simple models.

The purpose of the present paper is to determine the rôle of such parameters of the composite as the fibre aspect ratio, the volume fraction of fibres, and the distribution of fibres in a matrix. This will be done after a consideration of a shear model of a unidirectional fibrous composite with short rigid fibres under steady-state creep with a load applied in the direction of reinforcement. The resistance of a matrix to the tensile stress is assumed to be negligible, which corresponds to a large difference between the stiffness of the matrix and that of the fibres. The model material is composed of a number of simple shear cells, which are stretched under the condition of compatibility.

2. Creep of a Simple Shear Cell

2.1. The Plate-like Cell

We shall consider a simple shear cell composed of two rigid plates, which have length L in the x-direction, and area ABCD, which is a matrix attached to the plates in such a manner that the creep properties of the interface are not different from those of the matrix (fig. 1). The creep of the matrix is described by the usual equation [4]:

$$\epsilon = \epsilon_m \left(\frac{\sigma}{\sigma_m}\right)^m , \tag{1}$$

where ϵ is the tensile creep rate, σ is the tensile stress, and ϵ_m, σ_m, and m are constants, only two of which are independent. One can then write for pure shear of a matrix:

$$\eta = \epsilon_m \left(\frac{\tau}{\tau_m}\right)^m \tag{2}$$

where η is the shear strain rate, τ is the shear stress, and ϵ_m and m are constants equal to the

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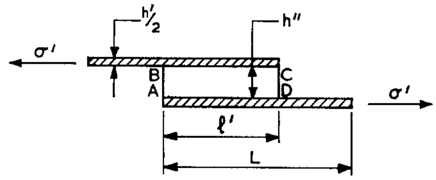

Figure 1 Longitudinal section of a simple plate-like cell.

same parameters as in equation 1. The Tresca-St. Venant condition of flow gives

$$\sigma_m = 2\tau_m \,. \tag{3}$$

The external load divided by the total transverse area of the two rigid plates will be denoted by σ'. Here and subsequently the values marked by one prime relate to reinforcing components; the values related to the matrix are marked by two primes; the ones related to the composite as a whole are not marked with primes. We shall call reinforcing components "fibres" independent of their real shape even if they are plates. The movement of fibres is assumed to be such that the directions of their longitudinal axes are not changed. The matrix is in a uniform state of simple shear and so the value of the shear stress at any point in the matrix will then be given by

$$\tau'' = \frac{\text{load}}{l'} = \frac{\sigma'^{h'}/2.2}{l'} = \tau'' = \frac{h'}{l'}\sigma' \,, \tag{4}$$

taking the shear cell in fig. 1 to have unit depth.

Using this value and equation 2 one can determine the shear rate η in the matrix and hence the rate of relative motion of plates, thus:

$$v = \eta h'' = 2^m \epsilon_m \left(\frac{\sigma'}{\sigma_m}\right)^m \left(\frac{h'}{l'}\right)^m h'' \,. \tag{5}$$

Introducing a value for the fibre volume fraction $V_f = h'/(h' + h'')$ we can rewrite equation 5 in the form:

$$v = 2^m \epsilon_m \left(\frac{\sigma}{\sigma_m}\right)^m \left(\frac{h'}{l'}\right)^m h' \, \frac{1/V_f - 1}{V_f^m} \,. \tag{6}$$

Here $\sigma = \sigma' V_f$ is stress, related to the total area of a transverse section of the cell.

2.2. The Cell of a Composite with Hexagonal Array of Fibres

Next we shall obtain a similar expression for a cell of another form, that shown in fig. 2. We shall be using this expression later in the calculation of the creep-rate of a model with a hexagonal net of fibres.

The shear stress distribution with regard to the

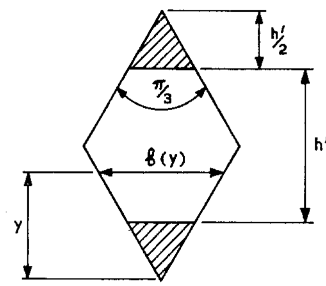

Figure 2 Transverse section of a shear cell to be used for the construction of a composite with hexagonal fibre distribution.

y-direction is now non-homogeneous; its value for the section that has size $b(y)$ can be found as follows, see fig. 2. The depth of the cell can be taken as l'. The shear stress in the matrix on a plane distance y from the fibre surface is

$$\tau_{(y)} = \frac{\text{load}}{A(y)} = \frac{\sigma' \times 2 \times \frac{1}{2} \times h'/2 \times h'/\sqrt{3}}{2(y/\sqrt{3}) \times l'} = \left(\frac{\sigma' h'}{2l'}\right)\frac{h'}{2y} \,,$$

$$\tau_{(y)} = \tau' \frac{h'}{2y} \text{ at } \tfrac{1}{2}h' \leqslant y \leqslant \tfrac{1}{2}(h' + h'') \tag{7}$$

where

$$\tau' = \tfrac{1}{2}\frac{h'}{l'}\sigma' \tag{8}$$

is the shear stress at the interface.

Equations 2, 3, 7, and 8 then determine the shear strain rate:

$$\eta_{(y)} = 2^{-m}\epsilon_m \left(\frac{\sigma'}{\sigma_m}\right)^m \left(\frac{h'}{l'}\right)^m \left(\frac{h'}{y}\right)^m \,. \tag{9}$$

The rate of relative motion of the two fibres will be

$$v = 2\int_{\frac{1}{2}h'}^{\frac{1}{2}(h'+h'')} \eta(y)\,\mathrm{d}y = \frac{\epsilon_m}{m-1}\left(\frac{\sigma}{\sigma_m}\right)^m = \left(\frac{h'}{l'}\right)^m h' \, \frac{1 - V_f^{\frac{m-1}{2}}}{V_f^m} \,. \tag{10}$$

Here

$$V_f = \left(\frac{h'}{h' + h''}\right)^2 .$$

3. Uniaxial Creep of a Non-homogeneous Rod

Before we start to consider the behaviour of a composite model composed of a number of the simplest cells with different values of overlap, l', let us consider the creep of a non-homogeneous rod which is a set of an infinite number of longitudinal elements loaded in parallel. The creep of each element will be governed by the equation

$$\epsilon = \epsilon_n \left(\frac{\bar{\sigma}}{\sigma_n}\right)^n, \tag{11}$$

where $\bar{\sigma}$ is the stress in the element, ϵ_n and n are constants, σ_n is not a constant and different elements have a different value for σ_n. Let us assume that there is some distribution $\phi(\sigma_n)$ of the value σ_n over the interval $(0, \sigma_n{}^*)$ so that

$$\int_0^{\sigma_n{}^*} \phi(\sigma_n)\, \mathrm{d}\sigma_n = 1 \cdot \tag{12}$$

If the rate of extension of all the elements is ϵ, we have

$$\bar{\sigma} = \sigma_n \left(\frac{\epsilon}{\epsilon_n}\right)^{1/n}.$$

Then the equation of equilibrium is

$$\int_0^{\sigma_n{}^*} \bar{\sigma}\phi(\sigma_n)\, \mathrm{d}\sigma_n = \sigma, \tag{13}$$

where σ is the average stress for a rod, and this gives

$$\epsilon = \epsilon_n \left(\frac{\sigma}{\langle\epsilon_n\rangle}\right)^n \tag{14}$$

where

$$\langle\sigma_n\rangle = \int_0^{\sigma_n{}^*} \sigma_n\phi(\sigma_n)\, \mathrm{d}\sigma_n \tag{15}$$

is the expectation of the value σ_n [5].

4. Creep of a Composite Model

4.1. Plate-like Reinforcement

We shall now consider a composite construction composed of plate-like cells of the symmetrical type ($l' = L/2$) as shown in fig. 3. Here q is the number of fibres within a given length and h' is the thickness of each fibre. The rate v of relative motion of the fibres under the condition of steady state creep is constant. The rate of relative displacement of two transverse sections, i.e. the rate of steady state creep of the model, will be:

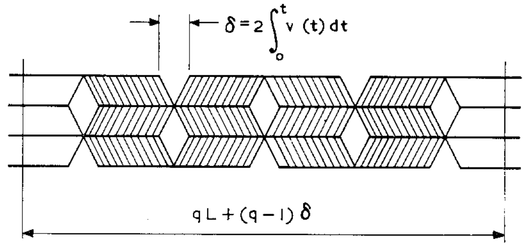

Figure 3 **Longitudinal section of the model with the ideal fibre distribution.**

$$\epsilon = (1 - 1/q)\frac{2v}{L} \approx \frac{2v}{L}, \tag{16}$$

since the number q is usually quite large.

Substituting equation 16 into equation 6 one gets

$$\epsilon = \epsilon_m \left(\frac{\sigma}{\sigma_m}\right)^m \frac{2^{2m+1}}{\rho^{m+1}} \frac{1/V_f - 1}{V_f{}^m}. \tag{17}$$

Here $\rho = L/h'$ is the aspect ratio of a fibre. We shall now estimate the influence of a non-symmetrical distribution of fibres in a longitudinal section. The parameter l' is assumed to have a value ranging from 0 to L with equal probability (fig. 4). The creep resistance of longitudinal

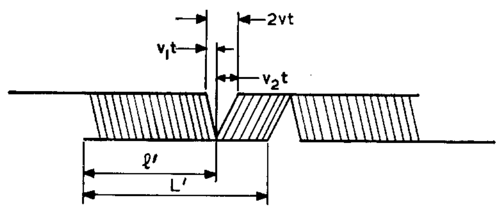

Figure 4 **A chain of the simple cells with non-symmetrical overlap of the fibres.**

elements composed of such cells will depend on the value l', and one can write, using equation 6:

$$2v = v_1 + v_2 = 2^m\epsilon_m \left(\frac{\bar{\sigma}}{\sigma_m}\right)^m h' \left[\left(\frac{h'}{l'}\right)^m + \left(\frac{h'}{L - l'}\right)^m\right] \frac{1/V_f - 1}{V_f{}^m}. \tag{18}$$

The meaning of the notations v_1 and v_2 is clear from fig. 4. The creep rate of a longitudinal element will be

$$\epsilon = \epsilon_m \left(\frac{\bar{\sigma}}{\sigma_m}\right)^m \frac{1/V_f - 1}{V_f{}^m} \left[\frac{1}{z^m} + \frac{1}{(1 - z)^m}\right] \times \frac{2^m}{\rho^{m+1}} \tag{19}$$

where $z = l'/L$ and $\bar{\sigma}$ is the stress in an element. Now the approach of section 3 can be used, and applying formulae 14 and 15, in which it is necessary to put

$$\sigma_n = \sigma_m \frac{1}{\left[\frac{1}{z^m} + \frac{1}{(1-z)^m}\right]^{1/m}} \quad (20)$$

and

$$\epsilon_n = \epsilon_m \frac{2^m}{\rho^{m+1}} \frac{1/V_f - 1}{V_f^m} \quad (21)$$

we obtain

$$\langle \sigma_n \rangle = \sigma_m J(m) \quad (22)$$

where

$$J(m) = \int_0^1 \frac{dz}{\left[\frac{1}{z^m} + \frac{1}{(1-z)^m}\right]^{1/m}}. \quad (23)$$

Finally instead of formula 17 for the regular distribution of fibres we have an expression for the creep rate for a model with fibres of random distribution in a longitudinal plane:

$$\epsilon = \epsilon_m \left(\frac{\sigma}{\sigma_m}\right)^m \frac{2^m}{\rho^{m+1}} \frac{1/V_f - 1}{V_f^m} [J(m)]^{-m} \cdot \quad (24)$$

Table I contains the values $J(m)$ for some values of the parameter m.

TABLE I

m	1	2	4	6	∞
$J(m)$	0.167	0.215	0.239	0.245	0.250

The situation here is similar to that which emerges in the solution of problems of steady state creep when one uses a creep law in the form of a power function [4]: the consideration of a stress state corresponding to that for an ideally plastic material ($m = \infty$) will not contribute a large error in the estimation of the load bearing capacity of a structure, if the power m has a real value $\geqslant 3$. In our case it is reasonable to assume $J(m) = J(\infty) = \frac{1}{4}$. A comparison of formulae 17 and 24 gives the relation of the stress for the random distribution of fibres (σ) and the ideal that (σ_i):

$$\sigma_i/\sigma = 2^{1-1/m}. \quad (25)$$

Proceeding in a similar manner one can show that small local fluctuations of the volume fraction V_f, which correspond to a non-regular structure in a transverse plane, have comparatively little effect on the creep-rate of a composite.

4.2. Hexagonal Fibre Array

Next we shall construct a model of a composite having as components the elemental cells shown in fig. 2. We shall make the model such that, situated at the nodes of the hexagonal plane lattice, will be the fibres of hexagonal form and of length L. The junctions between the longitudinal surfaces of fibres will be assumed to be welded in a rigid way. The distribution of fibres in a longitudinal section is such that the distribution function of the value l' is one of equal probability.

Now using equation 10 one can write a chain of expressions, which are similar to 18 to 24 and finally the expression for the creep-rate of a composite with a hexagonal net of fibres can be obtained in the form:

$$\epsilon = \frac{\epsilon_m}{m-1} \left(\frac{\sigma}{\sigma_m}\right)^m \frac{1}{\rho^{m+1}} \frac{1 - V_f^{\frac{m-1}{2}}}{V_f^m} [J(m)]^{-m}. \quad (26)$$

To judge the efficiency of the reinforcement of a creeping matrix with short rigid fibres let us introduce the parameter of hardening $k_{max} = \sigma_c/\sigma_M$ as a ratio of the average composite stress (σ_c) to the stress σ_M for a non-reinforced matrix which would produce the same creep-rate.

From equations 26 and 1 we obtain

$$k_{max} = (m-1)^{1/m} J(m) \frac{V_f}{(1 - V_f^{\frac{m-1}{2}})^{1/m}} \rho^{1+1/m}. \quad (27)$$

5. The Limits of Applicability

Here we shall trace the limits for the values of ρ and V_f within which the relationships obtained for the hexagonal net of fibres can be applied. (Similar expressions for a plate-like reinforcement can be obtained in the same way.) The assumption concerning the rigidity of the fibres is valid, as the rigidities of a matrix and fibres usually have different orders of magnitude. However, real fibres loaded by increasing stress can either rupture in a brittle manner or start to creep, if the temperature is high enough.

5.1. The First Critical Value of the Aspect Ratio

In the case of brittle fibres we shall define the first critical value ρ^* of the aspect ratio of fibre, as the value of ρ coinciding with the boundary between the interval for ρ, where formula 26 is still valid, and the interval where there are probably some fibres which are broken. The condition for the first fibres to rupture is when the most highly stressed fibres reach their ultimate strength. Since different longitudinal chains have different stiffnesses, the chain stresses vary. The fibre stress depends upon the relative location of the six fibres surrounding each fibre. The most highly stressed fibre will be that which is surrounded by symmetrical cells only ($z = \frac{1}{2}$). The probability of such a situation can be calculated, but to estimate ρ^* it is sufficient to know the value of the maximum stress ($\sigma'_{\max}$).

It can be shown that

$$\frac{\sigma'_{\max}}{\sigma'} = \frac{\sigma_{n\,\max}}{\langle\sigma_n\rangle}, \tag{28}$$

where σ' is an average fibre stress in a transverse section of the specimen. But since from equation 20

$$\sigma_{n\,\max} = \frac{\sigma_m}{2^{1+1/m}}$$

when $z = \frac{1}{2}$, we obtain by taking into account equation 22:

$$\sigma'_{\max} = \frac{\sigma'}{2^{1+1/m}J(m)}. \tag{29}$$

An average fibre stress σ' at the creep rate ϵ which is caused by the stress σ_M applied to a non-reinforced matrix can be obtained from equation 27 at $\rho < \rho^*$:

$$\sigma' = (m-1)^{1/m}J(m)\frac{\rho^{1+1/m}}{(1 - V_f^{\frac{m-1}{2}})^{1/m}}\sigma_M \cdot \tag{30}$$

Now we obtain from equations 29 and 30 the maximum value $\sigma'_{\max}$ of fibre stress and assuming it to be equal to the ultimate strength σ_*' of the fibre, we can write the expression for the critical value of ρ as:

$$\rho^* = 2\left(\frac{\epsilon_m}{\epsilon}\right)^{\frac{1}{1+m}}\left(\frac{\sigma_*'}{\sigma_m}\right)^{\frac{m}{1+m}}\left[\frac{1 - V_f^{\frac{m-1}{2}}}{m-1}\right]^{\frac{1}{1+m}}. \tag{31}$$

The significance of this value of the aspect ratio is that at $\rho > \rho^*$ the rate of hardening is not so high as at $\rho < \rho^*$, and some fibres fail because they cannot sustain the stress which can be transferred to them by the creeping matrix. A further stress redistribution takes place and the dependence of the creep-rate of a composite, given by equation 26, which contains component $\rho^{-(1+m)}$ is no longer valid. This situation is illustrated by fig. 5.

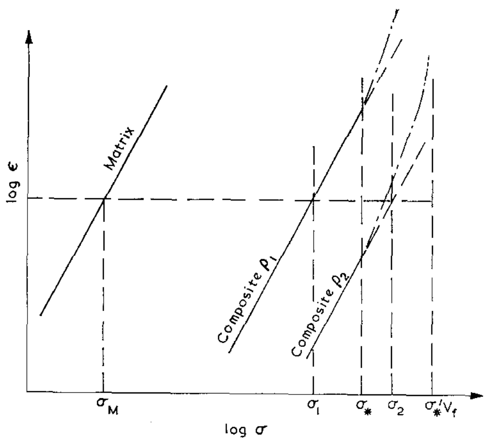

Figure 5 Schematic diagram for the creep behaviour of a fibrous composite with brittle fibres, V_f = const, $\sigma_* = \sigma_*' V_f\, 2^{1+1/m} J(m)$, (from equation 29) $\sigma_1/\sigma_M = k_{\max} > \sigma_2/\sigma_M$, $\rho_1 < \rho^* < \rho_2$.

5.2. The Second Critical Value of the Aspect Ratio

The model considered also does not work when the fibre stress is large enough to cause an appreciable creep-rate of the fibres. Here we sketch the method of estimating the second critical value ρ^{**} such that at $\rho > \rho^{**}$ the contribution of the fibre creep is larger than that of the shear creep

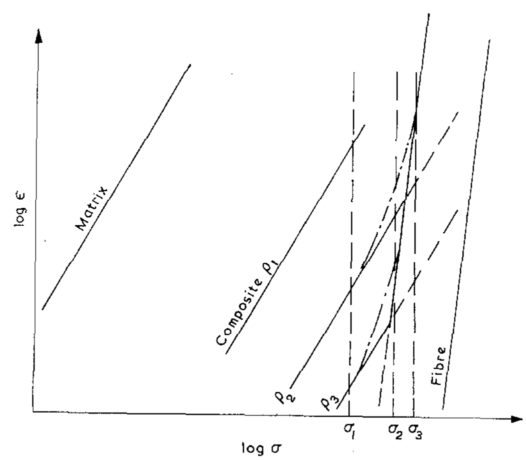

Figure 6 Schematic diagram for the creep behaviour of a fibrous composite with creeping fibres, V_f = const. For the stress σ_1: $\rho_1 < \rho_2 < \rho_3 < \rho^{**}$, for σ_2: $\rho_2 < \rho^{**} < \rho_3$, for σ_3: $\rho_1 < \rho^{**} < \rho_2$.

of a matrix. Hence at values of the aspect ratio which are much larger than ρ^{**} the fibres can be assumed to be continuous. For a continuous fibrous composite, neglecting a contribution of the matrix and a radial interaction of the fibre and the matrix, one can have:

$$\epsilon = \epsilon_f \left(\frac{\sigma}{\sigma_f}\right)^n \frac{1}{V_f^n} . \qquad (32)$$

Here $\epsilon_f = \epsilon_m$, σ_f and n are constants for a particular fibre material. Defining the value ρ^{**} as that which gives equal creep rates according to equations 26 and 32 ,we obtain:

$$\rho^{**} = \left[\frac{(\sigma/\sigma_m)^m}{(\sigma/\sigma_f)^n}\right]^{\frac{1}{1+m}} \frac{V_f^{\frac{n-m}{m+1}} \left(1 - V_f^{\frac{m-1}{2}}\right)^{\frac{1}{1+m}}}{[J(m)]^{m/1+m}(m-1)^{1/1+m}} . \qquad (33)$$

For $J(m) = \frac{1}{4}$ and for normal values of V_f and m:

$$\rho^{**} \cong 4 \left[\frac{\left(\frac{\sigma}{\sigma_m}\right)^m}{\left(\frac{\sigma}{\sigma_f}\right)^n}\right]^{\frac{1}{1+m}} V_f^{\frac{n-m}{m+1}} . \qquad (34)$$

It will be noticed that the expression in the brackets is the ratio of the creep rate of the matrix to that of the fibre at a stress equal to the average composite stress.

The relationships are illustrated in fig. 6. Here a change in behaviour of a composite from the short fibre situation to that of continuous fibres is traced schematically by dash-dot lines. These lines correspond to the behaviour of a composite in which the contribution of the fibre creep is of the same order of magnitude as that of the matrix creep.

Generally we shall define as a composite with shortfibres, one that satisfies both the inequalities

$$\rho < \rho^* \text{ and } \rho < \rho^{**} .$$

When one of those inequalities is not satisfied, we are dealing with a composite with long fibres.

5.3. The Limits for Volume Fraction

At sufficiently small values of the volume fraction of fibres the carrying capacity of a matrix for tensile stress should not be neglected. It is possible to find approximately such a minimum volume fraction $V_{f\,min}$, so that at $V_f = V_{f\,min}$ the creep rate of a matrix without reinforcement by fibres (1) and the creep rate of the model (26) are equal. The comparison of these values when V_f is small and m is sufficiently large gives:

$$V_{f\,min} = \frac{1}{(m-1)^{1/m}} \frac{1}{\rho^{1+1/m}} \frac{1}{J(m)} . \qquad (35)$$

At high volume fractions the model is also not valid. Bearing in mind the comparison of the model with a real composite containing circular fibres, one may find the value $V_{f\,max}$ corresponding to the close-packed arrangements of fibres, then $V_{f\,max} = 0.91$.

5.4. The Upper Limit of Carrying Capacity of a Composite

The results obtained should be considered as the lower limit of carrying capacity of a composite: the stress after these calculations is less than that given by an exact solution, because we have assumed some free surfaces within the volume of a composite to exist and have also neglected the transverse interactions between separate chains of simple cells.

The upper limit of the carrying capacity of the composite with the same free surfaces could be obtained after consideration of a kinematically possible stress field. Suppose the stress distribution through the set of simple cells is such that the rate of relative motion of two fibres in each cell is constant. Then $v = v_1 = v_2$, and from equations 10 and 16 we have, for a hexagonal array of fibres, the cell's stress for a given creep rate ϵ, and a cell with characteristic size z:

$$\sigma(z) = \left(\frac{\epsilon}{\epsilon_m}\right)^{\frac{1}{m}} \left(\frac{m-1}{2}\right)^{\frac{1}{m}} \rho^{1+\frac{1}{m}} \times \frac{V_f}{\left[1 - V_f^{\frac{m-1}{2}}\right]^{1/m}} \sigma_m z . \qquad (36)$$

Having the average stress σ in a transverse section for the distribution of the equal probability for a value z

$$\sigma = \int_0^1 \sigma(z)\, dz \qquad (37)$$

we obtain finally

$$\epsilon = \frac{2^{m+1}}{m-1} \epsilon_m \left(\frac{\sigma}{\sigma_m}\right)^m \frac{1}{\rho^{1+m}} \frac{1 - V_f^{\frac{m-1}{2}}}{V_f^m} . \qquad (38)$$

The ratio of the upper limit of stress σ_+ from equation 38 and the lower limit σ_- from equation 26 for a given creep rate

$$\sigma_+/\sigma_- \cong 2^{1-1/m} \quad (39)$$

is quite high. But more exact and more sophisticated analysis of this highly simplified composite model can hardly be justified. The conception of chains with fixed parameters is an assumption which simplifies drastically the analysis but could give an uncertain error.

Bearing this in mind we shall compare in the next section the results of the experiment with the theoretical prediction of the lower limit of composite stiffness.

6. Comparison with Experiments

There are few experimental results on the creep of discontinuous fibrous composites. The only available work [2] contains some results of experiments carried out with the composite Ag-W with a volume fraction of the tungsten wire $V_f = 0.4$ and ρ equal to 30 and 60. The temperature in these experiments was 400 to 600° C. In this work there is not enough information about the creep properties of the matrix used. Consequently the value of parameter m which will be used now has been taken from the work by Price [6] for silver with a purity of 99.99 %, and m for the temperature $T = 600°$ C has been obtained by extrapolation. The data on the creep properties of tungsten at $T = 600°$ C have been obtained from the work by Harris and Ellison [7] by extrapolation. All these data are collected in table II.

TABLE II

T, °C	σ_*', kgf/mm²	n	σ_f, kgf/mm²	m	σ_m, kgf/mm²	ϵ_m, h^{-1}
400	183	—	—	6.00	1.88	10^{-4}
600	167	53	145	4.50	0.66	10^{-4}

These data have been used for calculations of some parameters of the composites tested by Kelly and Tyson [2]. The results of the calculations are shown in table III.

TABLE III

T,°C	400	600
k_{max} at $\rho = 30$	8.10	8.57
k_{max} at $\rho = 60$	18.2	—
ρ^* at $\epsilon = 10^{-3}\,h^{-1}$	58	62
ρ^* at $\epsilon = 10^{-4}\,h^{-1}$	80	
ρ^{**} at $\sigma = 10$ kgf mm^{-2}	—	605
$V_{f\,min}$ at $\rho = 30$	0.058	0.125
$V_{f\,min}$ at $\rho = 60$	0.024	

The test results at the temperatures 400 and 600° C for the matrix (few points) and the composite with $\rho = 30$ are plotted in fig. 7.

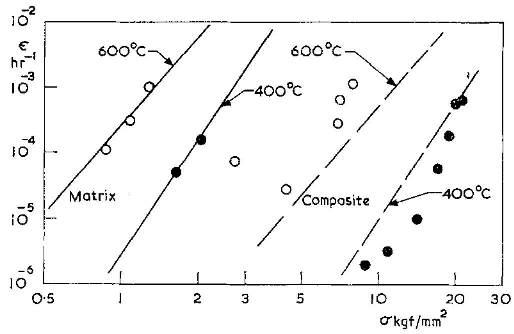

Figure 7 Comparison of the experimental data by Kelly and Tyson for Ag-W composite with $\rho = 30$. Open points – for 600° C, closed points – 400° C. – – – Results of calculations on the present method.

Theoretical lines are also shown. At 400° C the fit between the theoretical prediction and experimental data is quite good. The calculated curve gives stresses about 10 to 15% lower than the experimental results; this can be understood because the model must give a low estimation of the composite stress for a given creep rate. The departure of experimental points in the opposite direction at $T = 600°$ C can be caused by weakening of the interface as a consequence of diffusion of oxygen through the matrix and subsequent oxidation of the tungsten wire. This phenomenon was noticed by the authors [2].

To compare the experimental data obtained at 400° C for a composite with $\rho = 60$ is difficult because the value of the aspect ratio in this case is very close to the first critical value (table III), and therefore experimental points may be on the dash-dot line portion of curve $\epsilon - \sigma$ drawn in fig. 5. It is interesting to notice that the authors in their work observed that a small fraction of fibres was broken. This is quite in agreement with the model presented here. It will be noticed also that the description of the creep properties within a vast range of stress very often demands that equation 1 be changed in such a way that the exponent m is no longer a constant. It may take different values in different intervals of stress. In this case it is necessary to modify the calculations in the appropriate manner.

7. Conclusions

The behaviour of the simple shear model of a composite material reinforced with short rigid fibres has been analysed under conditions of steady-state creep. The analysis makes it possible to estimate the creep-rate of a fibre composite, and to determine its critical parameters. The comparison with the only set of experimental data available shows good enough agreement between the creep properties measured in the experiment and predicted values.

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List of main symbols

V_f = volume fraction of fibres in composite.
ρ = aspect ratio of a fibre
L = length of a fibre.
h' = transverse size of a fibre.
h'' = interfibre spacing.
m, σ_m, ϵ_m = constants for creep for a matrix material.
n, σ_f, ϵ_f = constants of creep for a fibre material, $\epsilon_f = \epsilon_m$.
v = rate of relative motion of two fibres.
σ_*' = ultimate strength of a fibre.
ρ^* = the first critical value of aspect ratio.
ρ^{**} = the second critical value of aspect ratio.

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